SEM - IV Date :- 18/2/25

MJC PHY05 Unit :- 02

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Numerical Problems :-

Q3. Solve $\sin x \frac{dy}{dx} + y\cos x = x \sin x$

Soln:- $\frac{dy}{dx} + y \cot x = x$

$\therefore IF = e^{\int \cot x\, dx} = e^{\ln(\sin x)} = \sin x$

$\therefore y \sin x = \int x \sin x\, dx + C$

$= -x\cos x + \sin x + C$

$\Rightarrow (y-1)\sin x + x\cos x = C$

Q4. Solve $x\frac{dy}{dx} - y = (x-1)e^{x}$

Soln:- Dividing by x, we get

$\frac{dy}{dx} - \frac{y}{x} = \frac{(x-1)}{x} e^{x}$

$\therefore IF = e^{-\int \frac{dx}{x}} = e^{-\ln x} = \frac{1}{x}$

$\Rightarrow \frac{y}{x} = \int \left(\frac{x-1}{x^2}\right) e^{x} dx$

$= \int \left(\frac{1}{x} - \frac{1}{x^2}\right) e^{x} dx$

Consider $\int \frac{e^x}{x} dx$

$$\int \frac{e^x}{x} dx = \frac{1}{x} e^x + \int \frac{1}{x^2} e^x dx$$

$$\Rightarrow \int \left(\frac{1}{x} - \frac{1}{x^2}\right) e^x dx = \frac{e^x}{x}$$

Thus $\frac{y}{x} = \frac{e^x}{x} + c$ where c is an arbitrary Constant.

Q(5.) Solve $x dy - \{y + xy^2(1 + \ln x)\} dx = 0$

$x dy - y dx = xy^2(1 + \ln x) dx$

Dividing by y^2

$$\frac{x dy - y dx}{y^2} = x(1 + \ln x) dx$$

$$\Rightarrow -d\left(\frac{x}{y}\right) = x(1 + \ln x) dx$$

On integration, we get

$$-\frac{x}{y} = \int x(1 + \ln x) dx + c$$

$$\Rightarrow -\frac{x}{y} = \frac{x^2}{2}(1 + \ln x) - \frac{x^2}{4} + c$$

——— x ———